**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

from scipy import stats

P(X>50) = 1 - stats.norm.cdf(50, loc = 45, scale = 8)

**0.26598552904870054**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**A)**

P(x>44) = 1 – P(x<44)

= (1 - stats.norm.cdf(44, loc = 38, scale = 6)).round(2)

= 0.16

400\*0.16 = 64

P(38<x<44) = P(x<44) – p(x<38)

= (stats.norm.cdf(44, loc = 38, scale = 6) - stats.norm.cdf(38, loc = 38, scale = 6)).round(2)

= 0.34

400\*0.34 = 136

Therefore, A is FALSE

**B)**

P(x<30) = (stats.norm.cdf(30, loc = 38, scale = 6)).round(2)

= 0.09

400\*0.09 = 36

Therefore, B is TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Solution:**

As both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,σ12+σ22). And 2 X1 will just scale the normal distribution by 2 times.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**Solution:**

Range for 99% of data lies between 3rd std of Mean

Mean=100, std=20

(100±3\*20) = (100-60, 100+60)

= (40,160)

Verification:

P(a<X<b) = 0.99

P(X<b) – p(X<a) = 0.99

from scipy import stats

stats.norm.cdf(x=151.5, loc=100, scale=20) - stats.norm.cdf(x=48.5, loc=100, scale=20)

0.9899759913364774

(48.5, 151.5)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Solution:**

from scipy import stats

Mean = 5+7

Mean

12

S\_d **=** 3**+**4

S\_d

7

mini, maxi**=** stats.norm.interval(alpha**=**0.95, loc**=**12, scale**=**7)

print('Range = ',(mini**\***45).round(2),(maxi**\***45).round(2))

Range = -77.39 1157.39

1. Specify the 5th percentile of profit (in Rupees) for the company

**Solution:**

X = Mu + Z \* sigma

Z value at 5% = -1.645

X = (12 + (-1.645\*7))\*45

= 21.824999999999974 Million Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Solution:**

stats.norm.cdf(0,5,3)

0.0477903522728147

stats.norm.cdf(0,7,4)

0.040059156863817086

Division 1 has larger probability of making a loss in a given year than Division 2